CS422 Principles of Database Systems
Multivalued Dependency
Chengyu Sun
California State University, Los Angeles

Motivational Example

<table>
<thead>
<tr>
<th>drinker</th>
<th>address</th>
<th>beerLiked</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sue</td>
<td>123 Main St.</td>
<td>Bud</td>
</tr>
<tr>
<td>Sue</td>
<td>321 State St.</td>
<td>Pete's Ale</td>
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Motivational Example Questions:
◆ FD?? Keys??
◆ 3NF?? BCNF??
◆ Is this a good design??

A New Form of Redundancy

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A New Form of Redundancy

Any combination of address and beerLiked for Sue is a valid tuple

Multivalued Dependency (MVD)

A Multivalued Dependency (MVD) $A \quad B$ is an assertion that if two tuples of a relation agree on all the attributes of $A$, then their components in the set of attributes $B$ may be swapped, and the result will be two tuples that are also in the relation.

In the drinkers example:

$A?? B?? C=R-AB??$

$?? ??$

A Couple of Observations about MVD

◆ MVD characterizes the case where one relation tries to represents more than one many-to-many relationships.
◆ MVD vs. FD (why it's called multivalued dependency)

Trivial MVD

$A \quad B$

Trivial MVD

◆ MVD is trivial if
  $B \subseteq A$, or
  $A \cup B = R$
Proof by Chase

- Given a set of FDs and MVDs $D$, does another dependency $d$ (FD or MVD) follow from $D$?
- Procedure
  - Start with the hypotheses of $d$, and treat them as "seed" tuples in a relation
  - Apply the given dependencies in $D$ repeatedly
    - If we apply an FD, we infer equality of two symbols
    - If we apply an MVD, we infer more tuples
  - If we infer the conclusion of $d$, we have a proof; otherwise we have a counter-example

From Jun Yang's lecture notes at http://www.cs.duke.edu/~junyang

Proof by Chase Example

- In $R(A, B, C, D)$, does $A$ $B$ and $B$ $C$ imply $A$ $C$?

<table>
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<tr>
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<th>D</th>
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<tr>
<td>$a$</td>
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<td>$c_1$</td>
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Have

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Need

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Union and Decomposition

- Union: if $A$ $B$ and $A$ $C$, then $A$ $B$ $C$
  - Proof??
- Decomposition rule no longer holds
  - Counter-example??

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Fourth Normal Form (4NF)

- A relation R is in 4NF if for every nontrivial MVD \( A \rightarrow\rightarrow B \), \( A \) is a super key.

Decompose into 4NF

- Find a 4NF violation \( A \rightarrow B \)
- Decompose R into:
  - \( R_1 = A \cup B \)
  - \( R_2 = (R - AB) \cup A \)
- Repeat until all relations are in 4NF

4NF Decomposition Example

- Drinkers(name, addr, beerLiked, favBeer)
  - FD?? Key??
  - MVD??

4NF Decomposition vs. BCNF Decomposition

- In 4NF decomposition we do not compute \( A^+ \)
  - \( A^+ \) does not make sense for MVD
  - \( A \) \( (R \cap A) \) and \( A \cap A \)
- Inferring MVDs for the projections are very difficult
  - However, we can usually get by using the rules of transitivity, complementation, and intersection.

Exercise: Prove the Intersection Rule

- If \( A \rightarrow B \) and \( A \rightarrow C \), then \( A \rightarrow B \cap C \)

4NF vs. BCNF

- Why??